# Discussion 6 Worksheet

## Tangent Planes and Linear Approximations

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MATH 53 Multivariable Calculus

## 1 Tangent Planes

Find the tangent planes to the graphs of each of the following functions at an arbitrary point $(x_0, y_0, f(x_0, y_0))$.

1. $f(x, y) = x^2 + 2xy + y^2$
2. $f(x, y) = e^{xy}$.
3. $f(x, y) = \sin x$.

## 2 More Tangent Planes

1. Find an equation for the tangent plane to the graph of $f(x, y) = \cos(xy)$ passing through the point $(\pi/2, 1, 0)$.
2. Find a parametric equation for a line contained in the tangent plane you found in the previous problem. (Any line will suffice.)

## 3 Linear Approximations

1. Find the best linear approximation to each of the following functions near the corresponding input values.
   a) $f(x, y) = y^2 - x$ near the input $(3, 0)$.
   b) $g(x, y) = e^x \cos y$ near the input $(5, \pi/2)$.
   c) $h(x, y, z) = xyz$ near the input $(3, 0, 2)$.
   d) $p(x, y, z, w) = x^2 + y^2 + z^2 + w^2$ near the input $(0, 1, 0, -1)$.
2. Consider a differentiable function $f(x, y)$ with values given by the following table.

| | $x = 1.0$ | $x = 1.2$ |
|---|---|---|
| $y = 0.0$ | 5.2 | 5.4 |
| $y = 0.2$ | 6.0 | 6.2 |

   a) Find the best linear approximation to $f(x, y)$ near the input value $(1.0, 0.0)$.
   b) Use this linear approximation to compute approximate values for $f(1.0, 0.1)$, $f(1.1, 0.0)$ and $f(1.1, 0.1)$.

## 4 Implicit differentiation

1. Find $\partial z/\partial x$, $\partial z/\partial y$ and $\partial x/\partial y$ when $x, y$ and $z$ satisfy the relation $x^2 + y^2 + z^2 = 3xyz$.

2. (Challenge) Suppose that $x, y, z$ are related by an equation $F(x, y, z) = 0$ (this is the setup for implicit differentiation). Show that

$$\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} = -1$$

## 5 Challenge

1. Let $S$ be a sphere centered at the origin in $\mathbb{R}^3$, and consider any point $P$ on $S$. Show that the vector $\overrightarrow{OP}$ is orthogonal to the tangent plane to $S$ at $P$.

## 6 True/False

Supply convincing reasoning for your answer.

(a) T F The vector $\langle f_x(x_0, y_0), f_y(x_0, y_0), -1\rangle$ is orthogonal to the tangent plane of the graph of $f(x, y)$ through the point $(x_0, y_0, z_0)$.

(b) T F Any tangent plane to a graph must meet that graph in exactly one point.

**Note:** These problems are taken from the worksheets for Math 53 in the Spring of 2021 with Prof. Stankova.